Systematics of *K* and *L* x-ray satellite spectra

V. Horvat and R. L. Watson

Multiple inner-shell ionization of an atom in a heavy-ion collision and the vacancy rearrangement that follows are fundamental processes that so far have not been adequately described theoretically. The only fundamental theory of multiple inner-shell ionization whose results are widely available is the geometrical model [1]. It predicts that in central collisions (i.e. collisions restricted to small impact parameters) the average fraction of vacancies produced in the shell having principal quantum number *n* is given by [1]

$$p_n = X_n /[4.2624 + X_n^2 (1 + 0.5\exp(-X_n^2/16))] \ , \tag{1}$$

where

$$X_n = 4V[G(V)]^{1/2} Z_1 \alpha c/(n v_1) \tag{2}$$

is referred to as a universal variable. In eq. (2), Z_1 is the projectile atomic number, α is the fine structure constant *c* is the speed of light in vacuum, v_1 is the projectile speed, *n* is the principal quantum number of the spectator vacancies, $V = v_1/v_2$ is the scaled projectile speed (v_2 is the average speed of an electron having principal quantum number *n*), and *G*(*V*) is the binary encounter approximation (BEA) scaling function [2].

However, we found [3] that this result greatly overestimates the measured average number of *L*- and *M*-shell vacancies produced in central heavy-ion collisions and, therefore, cannot be relied upon for accurate predictions. Nevertheless, based on measurements of *K*α x-ray spectra involving a large variety of thick solid targets, projectiles, and collision energies, we have established [3] that the measured values of the apparent average fraction of *L* vacancies at the time of *K* x-ray emission (p_L^x) define a “universal” curve when plotted as a function of X_2. Furthermore, it was found [3] that the available data can be well represented by

$$p_L^x = a_2/[1 + (b_2/X_2)^{c_2}], \tag{3}$$

using $a_2 = 0.537 \pm 0.006$, $b_2 = 2.11 \pm 0.08$, $c_2 = 2.02 \pm 0.03$.

This analysis was subsequently applied to the spectra of target *L* x rays [4]. It was found that a formula similar to eq.(3) can be used to describe the apparent average fraction of *M* vacancies at the time of *L* x-ray emission;

$$p_M^x = a_3/[1 + (b_3/X_3)^{c_3}], \tag{4}$$

where $a_3 = 0.530 \pm 0.012$, $b_3 = 1.507 \pm 0.044$, and $c_3 = 2.15 \pm 0.12$. This finding adds considerable weight to the significant role the geometrical model variable plays in the description of multiple ionization by heavy ion impact. These results are important because they provide a means of accurately predicting the characteristics of *K* and *L* x-ray satellite distributions for other collision systems, regardless of projectile energy, projectile atomic number, and target atomic number.

The two curves defined by eqs.(3,4) are shown in Figure 1. The one representing $p_M{}^x$ as a function of X_3 [eq. (4)] lies above the one representing $p_L{}^x$ as a function of X_2 [eq. (3)]. The difference between the two sets of values reaches a maximum of 0.089 ± 0.017 at $X = 1.9$. However, at large X the difference is statistically insignificant ($a_3 - a_2 = 0.007 \pm 0.013$). The maximum difference is significantly larger than the standard deviation of the residuals (0.013) obtained in the fit to the measured data points. This indicates that there may be some deviation from the universal scaling predicted by the geometrical model at the intermediate values of the universal variable. However, if the two curves are plotted as a function of X/b (as shown in Figure 2), the maximum difference between them is less than 0.009.

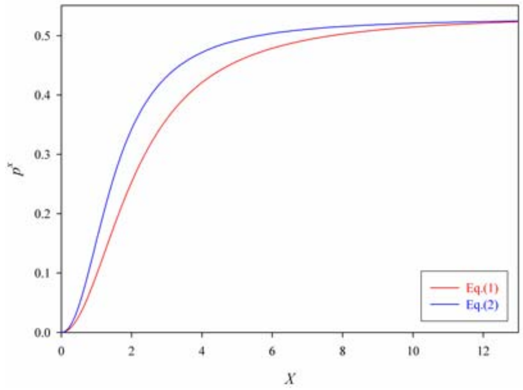

Figure 1. Apparent average fraction of *L*-shell [eq.(3)] and *M*-shell vacancies [eq.(4)] at the time of *L* and *M* x-ray emission, respectively, as a function of the universal variable.

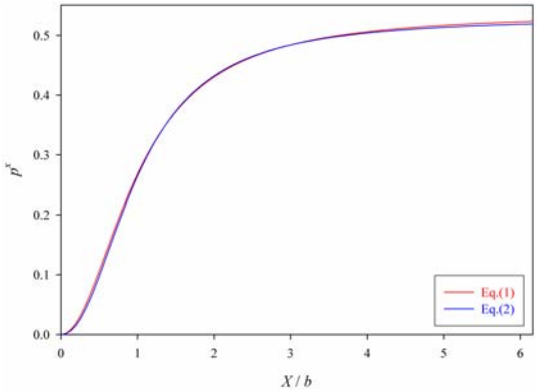

Figure 2. Apparent average fraction of *L*-shell [eq.(3)] and *M*-shell vacancies [eq.(4)] at the time of *L* and *M* x-ray emission, respectively, as a function of the universal variable divided by the empirical parameter *b*.

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